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ON THE EFFECT OF THE INTERNAL VISCOSITY MECHANISM ON THE PERFECTLY PLASTIC BEHAVIOR OF MATERIALS*

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A model is constructed for the mechanical behavior of a plastic material, exhibiting viscosity properties within a restricted range of deformation rates. A solution of the problem of behavior of a thick-walled pipe under internal pressure is given within the framework of this model.

The author of /1/ dealt with the influence of prehardening on the perfectly plastic flow of a material. In this case the elasticity mechanism is internal (Fig.1a). Below we consider the effect of internal viscosity mechanism on the perfectly plastic behavior of a material. The corresponding mechanical schemes are shown in Fig.1b and c. The scheme shown in Fig.1b assumes the continuity of the force adherence between the elements of dry friction, i.e. the incorporation of Maxwell-type viscosity element, and Fig.1c assumes the continuity of the kinematic adherence, i.e. the incorporation of the Kelvin-type viscosity element. It is shown that the internal viscosity element leads, in the case shown in Fig.1b, to accumulation of internal microdeformations in the course of a flow of a perfectly plastic material with yield point $k_1 + k_2$. In the second case (Fig.1c) we have the usual viscoelastic Bingham body. The influence of the viscosity on the mechanical behavior of plastic bodies was studied in /2,3/. In terms of the notation used in /3/ the material corresponding to the scheme depicted in Fig.1a is denoted by the index Pep, that in Fig.1b by $Pv^\circ p$ and the one in Fig.1c by Pv^*p.

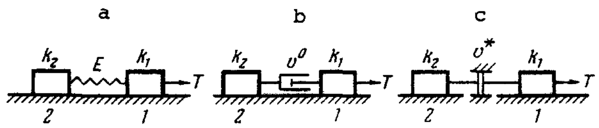

Fig.1

1. We begin by considering the model $Pv^\circ p$ (Fig.1b). The motion of the first dry friction element begins when the external force T reaches the limiting value k_1. If the rate of motion of the first friction element is infinitesimal, then the viscosity element offers no resistance and the corresponding model behaves like a perfectly plastic body with yield point equal to k_1 (line ab in Fig.2). When the rate of motion increases, the viscous resistance grows and the corresponding model behaves like the viscoplastic Bingham body until the internal forces do not succeed in bringing into motion the second internal friction element (line ac in Fig.2). When the loading rate is sufficiently fast, the internal friction element comes into operation and the corresponding model behaves like a perfect plastic body with yield point equal to $k_1 + k_2$ (line a_1b_1 in Fig.2). Thus we see that under an infinitely slow loading rate the model in question behaves like a perfectly plastic body with yield point equal to k_1, and under an instantaneous loading, like a perfectly plastic body with yield point equal to $k_1 + k_2$.

Let us denote by σ_{ij} the real stress tensor, and by s_{ij} the internal stress tensor corresponding to the force transmitted through the friction element to the second friction element. Denoting the deviators of the corresponding tensors by a prime and assuming for simplicity that the material is incompressible, we write the condition of plasticity in the form

$$(\sigma_{ij}' - s_{ij}')(\sigma_{ij}' - s_{ij}') = k_1^2, \quad k_1 = \text{const} \tag{1.1}$$

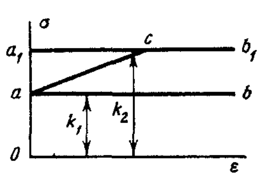

Fig.2

where k_1 denotes the yield point of the material corresponding to the limiting dry friction of the first friction element. In accordance with the associated flow rule, we have

$$\varepsilon_{ij} = \lambda(\sigma_{ij}' - s_{ij}'), \quad \lambda = \frac{1}{k_1}\sqrt{\varepsilon_{ij}\varepsilon_{ij}} \tag{1.2}$$

*Prikl.Matem.Mekhan.,Vol.47,No.3,pp.524-527,1983 where ε_{ij} is the plastic deformation rate tensor. For the internal plasticity element corresponding to the second friction element we have

$$s_{ij}'s_{ij}' \leqslant k_2^2, \quad k_2 = \text{const} \tag{1.3}$$

Let us denote by $\varkappa_{ij}$ the internal microdeformations rate tensor corresponding to the rate of displacement of the second friction element. Then, according to (1.3) and the associated flow rule, we obtain

$$\varkappa_{ij} = \mu s_{ij}', \quad \mu = \frac{1}{k_2}\sqrt{\varkappa_{ij}\varkappa_{ij}} \tag{1.4}$$

We write the relations characterising the mechanical behavior of the friction element in the form (η is the coefficient of viscosity)

$$s_{ij}' = \eta\varepsilon_{ij}, \quad s_{ij}'s_{ij}' < k_2^2 \tag{1.5}$$

$$s_{ij}' = \eta(\varepsilon_{ij}' - \varkappa_{ij}), \quad s_{ij}'s_{ij}' = k_2^2 \tag{1.6}$$

and in case of (1.5) we have a viscoplastic Bingham body.

Below we consider the case when $s_{ij}'s_{ij}' = k_2^2$. The behavior of the model in question is described by the equations (1.1) – (1.4),(1.6). Eliminating the quantity $\varkappa_{ij}$ from (1.4) and (1.6), we obtain

$$s_{ij}' = \frac{1}{1+\eta\mu}\varepsilon_{ij} = \frac{1}{\mu^*}\varepsilon_{ij} \tag{1.7}$$

$$\mu^* = \frac{1}{k_2}\sqrt{\varepsilon_{ij}\varepsilon_{ij}}, \quad \mu = \frac{1}{k_2}\sqrt{\varepsilon_{ij}\varepsilon_{ij}} - \frac{1}{\eta}$$

Since $\mu \geqslant 0$, the last equation of (1.7) yields

$$\eta\sqrt{\varepsilon_{ij}\varepsilon_{ij}} \geqslant k_2 \tag{1.8}$$

The above relation determines, provided that the inequality is strict, the condition of accumulation of microdeformations. In the case of equality in (1.8) and we have $\mu = 0$ but from (1.4) it then follows that $\varkappa_{ij} = 0$. Then from (1.1), (1.2) and (1.7) we obtain

$$\sigma_{ij}' = \left(\frac{k_1}{\sqrt{\varepsilon_{ij}\varepsilon_{ij}}} + \eta\right)\varepsilon_{ij}, \quad \sqrt{\sigma_{ij}'\sigma_{ij}'} = k_1 + \eta\sqrt{\varepsilon_{ij}\varepsilon_{ij}}$$

If the general condition (1.8) holds, then (1.1), (1.2), (1.4) and (1.6) yield

$$\varepsilon_{ij} = \frac{\sqrt{\varepsilon_{ij}\varepsilon_{ij}}}{k_1+k_2}\sigma_{ij}', \quad \sigma_{ij}'\sigma_{ij}' = (k_1+k_2)^2 \tag{1.9}$$

Consequently, as was said before, according to (1.9), when the internal plasticity element is deformed, then the body behaves like a perfectly plastic body with yield point equal to $k_1 + k_2$. In the case of rigid mechanical adherence between the plasticity elements and with $\eta \to \infty$, we have, according to (1.6), $\varepsilon_{ij} = \varkappa_{ij}$. When $\eta \neq 0$, then the body whose outside behaves as perfectly plastic, accumulates internal microdeformations. It can be assumed that the growth of the microdeformations results in fracture of the material. The fracture criteria can e.g. be introduced in the form

$$\int\sqrt{\varkappa_{ij}\varkappa_{ij}}\,dt \leqslant K, \quad K = \text{const}$$

Let us determine the dissipation function

$$D = \sigma_{ij}'\varepsilon_{ij} \tag{1.10}$$

When condition (1.8) holds, then the dissipation function has, in accordance with (1.10) and the first relation of (1.9), the form

$$D = (k_1 + k_2)\sqrt{\varepsilon_{ij}\varepsilon_{ij}} \tag{1.11}$$

In case of equality in (1.8), from (1.11) it follows that

$$D = D^* = k_2(k_1 + k_2)/\eta$$

Thus the condition of accumulation of microdeformations can be written in the form

$$D > D^*$$

2. Consider the model Pv^*p (Fig.1c). In this case the internal stress tensor corresponding to the force acting between the first (second) friction element and the viscosity element will be denoted by $s_{ij(1)}$ (according to $s_{ij(2)}$). Then we have

$$(\sigma_{ij}' - s_{ij}')(\sigma_{ij}' - s_{ij}') = k_1^2, \quad \varepsilon_{ij} = \lambda(\sigma_{ij}' - s_{ij}') \tag{2.1}$$

$$s_{ij(2)}'s_{ij(2)}' = k_2^2, \quad \varepsilon_{ij} = \mu s_{ij(2)}'$$

$$s_{ij(1)}' - s_{ij(2)}' = \eta\varepsilon_{ij}$$

from which we obtain

$$\sqrt{\sigma_{ij}'\sigma_{ij}'} = k_1 + k_2 + \eta\sqrt{\varepsilon_{ij}\varepsilon_{ij}}$$

$$\sigma_{ij}' = \left(\frac{k_1+k_2}{\sqrt{\varepsilon_{ij}\varepsilon_{ij}}} + \eta\right)\varepsilon_{ij}$$

The above relations define the viscoplastic Bingham body.

3. We consider, as an example, the axisymmetric behavior of a thick-walled pipe acted upon by uniform internal pressure p, in the case of plane deformation. When viscoplastic flow of the material takes place (ρ and θ are polar coordinates),

$$\sigma_\theta - \sigma_\rho = \sqrt{2}\,k_1 + \eta(\varepsilon_\theta - \varepsilon_\rho) \qquad (3.1)$$

Let $u = u(\rho)$ denote the radial velocity. Then

$$\varepsilon_\rho = du/d\rho, \quad \varepsilon_\theta = u/\rho \qquad (3.2)$$

The equation of incompressibility yields

$$u = c/\rho, \quad c = c(t) \qquad (3.3)$$

and from (3.1)—(3.3) we obtain

$$\sigma_\theta - \sigma_\rho = \sqrt{2}\,k_1 + 2\eta\frac{c}{\rho^2} \qquad (3.4)$$

The equation of equilibrium now becomes

$$\frac{d\sigma_\rho}{d\rho} = \frac{\sqrt{2}\,k_1}{\rho} + 2\eta\frac{c}{\rho^3} \qquad (3.5)$$

Integrating (3.5) we obtain

$$\sigma_\rho = \sqrt{2}\,k_1 \ln\rho - \eta\frac{c}{\rho^2} + A, \quad A = A(t) \qquad (3.6)$$

and from (3.4) we have

$$\sigma_\sigma = \sqrt{2}\,k_1(\ln\rho + 1) + \eta\frac{c}{\rho^2} + A$$

Remembering that $\sigma_\rho = -p$ when $\rho = a$ and $\sigma_\rho = 0$ when $\rho = b$ (a and b denote the internal and external radius of the pipe respectively), we obtain from (3.6)

$$c = \left(p - \sqrt{2}\,k_1 \ln\frac{b}{a}\right)\frac{a^2b^2}{(b^2-a^2)\eta}, \quad A = \frac{pa^2 - \sqrt{2}\,k_1(b^2\ln b - a^2\ln a)}{b^2-a^2}$$

Increasing the internal pressure p leads to the appearance of a plastic zone in which

$$\sigma_\theta - \sigma_\rho = \sqrt{2}\,(k_1+k_2), \quad \sigma_\rho = \sqrt{2}\,(k_1+k_2)\ln\frac{\rho}{a} - p$$

The energy dissipation in

$$D = \sigma_\rho\varepsilon_\rho + \sigma_\theta\varepsilon_\theta = (\sigma_\theta - \sigma_\rho)\,\varepsilon_\theta$$

In the viscoplastic flow zone (3.4) holds, therefore

$$D = \left(\sqrt{2}\,k_1 + 2\eta\frac{c}{\rho^2}\right)\frac{c}{\rho^2} \qquad (3.7)$$

At the boundary separating the viscous and plastic zones

$$D = D^* \qquad (3.8)$$

Equations (3.7) and (3.8) yield an equation for determining the radius of the boundary separating the viscous and plastic zones. Clearly, when

$$\sqrt{2}\,k_1\ln(b/a) < p < \sqrt{2}\,(k_1+k_2)\ln(b/a)$$

then viscous and plastic zone both exist, while when

$$p = \sqrt{2}\,(k_1+k_2)\ln(b/a)$$

we have the plastic zone only.

REFERENCES

1. IVLEV D.D., Theory of Perfect Plasticity. Moscow, NAUKA, 1966.
2. BEREZHNOI I.A. and IVLEV D.D., On the influence of viscosity on mechanical behavior of plastic media. Dokl. Akad. Nauk SSSR, Vol.163, No.3, 1965.
3. IVLEV D.D., On the theory of composite media. Dokl. Akad. Nauk SSSR, Vol.148, No.1, 1963.

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